COMPUTATION OF THE VELOCITY PROFILE IN THE FLOW OF AN ANOMALOUSLY VISCOUS LIQUID IN CYLINDRICAL CHANNELS OF ARBITRARY CROSS SECTION

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A method is proposed for computing the velocity profile in the flow of an anomalously viscous liquid in cylindrical channels of arbitrary cross section as a function of the pressure drop, the dimensions of the transverse cross section, and the rheological characteristics of the liquid.

The theoretical investigation of the hydrodynamics of the flow of an anomalously viscous liquid in channels of noncircular transverse cross section is of considerable applied interest for a number of branches of chemical and power engineering. Such flows are extensively used in practice: production of profiled parts, flow of heat carrier in compact heat-transfer systems, and so forth.

Many investigators have investigated the problems of flow of anomalously viscous liquids in cylindrical prismatic channels. However, the results of the investigations known to us are valid only for the simplest forms of the transverse cross sections of the channels or only for particular types of rheological equations, which reduces their practical value. Furthermore, the flow-rate characteristics were investigated in these studies, which is of interest for the case of isothermal flow. In order to solve the heat-transfer problems in the flow of anomalously viscous liquids in noncircular channels it is necessary to determine the velocity profile, which, in turn, will determine the temperature distribution in the liquid flow.

In the present work an attempt is made to determine analytically the velocity profile in an anomalously viscous liquid, described by an arbitrary rheological equation, during its flow in a cylindrical channel of arbitrary cross section.

We consider a laminar steady-state flow of an anomalously viscous liquid in a cylindrical channel of arbitrary cross section (Fig. 1). The equation of motion and the continuity equation have the following forms in a Cartesian coordinate system:

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = -\Delta P = \text{const}, \tag{1}$$

$$\frac{\partial V}{\partial z} = 0. \tag{2}$$

The tangential stress components are given by

$$\tau_x = \mu \frac{\partial V}{\partial x}, \quad \tau_y = \mu \frac{\partial V}{\partial y}. \tag{3}$$

Assuming that the nature of the distribution of the shear stress along the cross section of the channel is the same in the anomalously viscous liquid as in the case of the flow of a viscous liquid, we have

$$\tau_x = 0.5\,\Delta P \frac{\partial U}{\partial x}, \quad \tau_y = 0.5\Delta P \frac{\partial U}{\partial y}. \tag{4}$$

Substituting (4) into (1) we obtain Poisson's equation:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -2. \tag{5}$$

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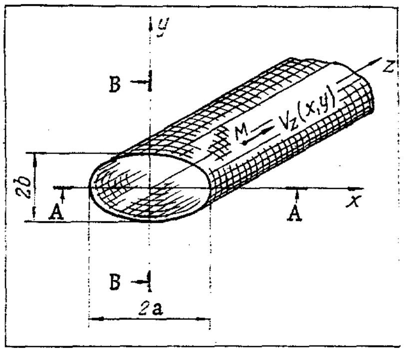

Fig. 1

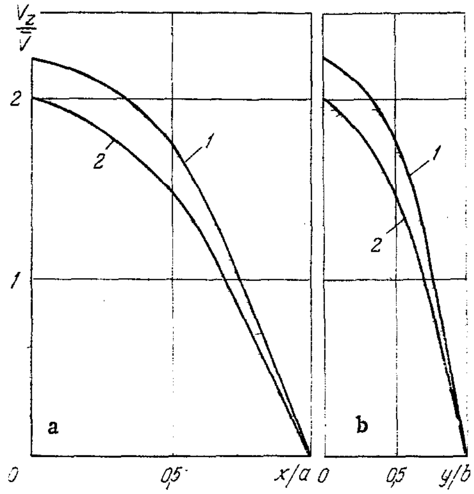

Fig. 2

Fig. 1. Schematic diagram of the flow in an elliptical channel.

Fig. 2. Computed velocity profiles in cross sections A–A and B–B for the flow of: 1) 4.75% aqueous solution of Na–CMC ($\varphi_0 = 0.298\ P^{-1}$; K = 0.123; n = 0.35); 2) Newtonian liquid (K = 0, n = 1) in an elliptical channel.

Thus, in the present formulation the problem of flow of an anomalously viscous liquid in a prismatic channel reduces to the solution of the Dirichlet problem for the function U. At the same time, Poisson's equation (5) describes also the stress function in the presence of torsion of a prismatic bar of the same cross section as the investigated channel. At present, a mathematical tool for solving this problem has been developed.

We define the velocity profile in the form of the function V = f(x, y) in the following way:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy. \tag{6}$$

With (3) and (4) taken into consideration, Eq. (6) becomes

$$dV = \varphi\tau_x dx + \varphi\tau_y dy = 0.5\Delta P\varphi\left(\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy\right). \tag{7}$$

Using an arbitrary rheological law,

$$\varphi = f(\tau^2) = f(\tau_x^2 + \tau_y^2) = f\left\{(0.5\Delta P)^2\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2\right]\right\}, \tag{8}$$

the velocity distribution in a cylindrical channel of arbitrary cross section can be written in the following general form:

$$V(x,\ y) = 0.5\Delta P\int f\left\{(0.5\Delta P)^2\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2\right]\right\}\left(\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy\right) + C. \tag{9}$$

It is obvious that the constant of integration in (9) is equal to the maximum value of the velocity V_{max}, which is equivalent to the boundary condition of adhesion of the liquid to the wall.

Formula (9) enables one to compute the velocity profiles in anomalously viscous liquids described by different rheological equations. For example, considering the flow of a liquid obeying law (10) in an elliptical channel,

$$\varphi = \varphi_0 + K\tau^m, \tag{10}$$

for which the function U is of the form [1]

$$U = \frac{a^2b^2}{a^2+b^2}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right), \tag{11}$$

We obtain the velocity distribution over cross sections A—A and B—B (Fig. 1) in the following form:

$$V(x,\ 0) = \frac{\Delta P}{2}\left\{\varphi_0 \frac{a^2b^2}{a^2+b^2}\left(1-\frac{x^2}{a^2}\right) + \frac{2K\Delta P^m}{(m+2)(a^2+b^2)^m a^2b^2}[(a^2b^2)^{m+2} - (x^2b^2)^{m+2}]\right\}, \tag{12}$$

$$V(0,\ y) = \frac{\Delta P}{2}\left\{\varphi_0 \frac{a^2b^2}{a^2+b^2}\left(1-\frac{y^2}{b^2}\right) + \frac{2K\Delta P^m}{(m+2)(a^2+b^2)^m a^2b^2}\left[(a^2b^2)^{m+2} - (y^2a^2)^{m+2}\right]\right\}. \tag{13}$$

The computed dimensionless velocity profiles over cross sections A—A and B—B are shown in Fig. 2 for the flow of a 4.75% aqueous solution of sodium carboxymethyl cellulose (CMC) in an elliptical channel (a = 5.5 mm, b = 2.75 mm) along with the computed velocity profiles for a Newtonian liquid (K = 0, n = 1). As is evident from (12) and (13), for the case of the Newtonian liquid they coincide with the well-known expressions [2].

The mean velocity required for constructing the dimensionless velocity profiles was determined in terms of the volume flow rate of the liquid, which was computed from the expression [3-5]

$$Q = \frac{1}{\Delta P}\iint \frac{\tau^2}{\mu}\, dxdy, \tag{14}$$

where

$$\tau^2 = \tau_x^2 + \tau_y^2 = \left(\frac{\Delta P}{2}\right)^2\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2\right].$$

The double integral (14) is easily evaluated by the cubic formulas with any degree of accuracy.

Thus, formula (9) enables one to compute the velocity profile for a channel with a cross section for which the solution of Poisson's equation is known. This means that with the methods of computation currently available the velocity profiles can be computed for cylindrical channels of completely arbitrary cross sections including multiply connected channels.

NOTATION

τ, shear stress; μ, effective viscosity; φ, fluidity; φ_0, fluidity as $\tau \to 0$; V, flow velocity; ΔP, pressure drop per unit length of the channel; τ_x, τ_y, shear-stress components; K, m, rheological equation constants; a, b, semiaxes of the ellipse; γ, shear-rate gradient; Q, volumetric flow rate of the liquid.

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